Chapter 1 : Real Number

Fundamental Theorem of Arithmetic

Every Composite Number can be expressed as a product of primes and this expression is unique,except the order in which it is written.

Composite Number = Product of Primes

Factor of a Number :

If a Number divides another number exactly, then the number which divides, is called a factor of that number.

Real Number

- **Rational** — Can be Written in p/q Form
  - If $q = 2^m \times 5^n$ → **Terminating** — If Prime Factor of Denominator Contains Only 2 & 5
    - $\frac{13}{8} = \frac{13}{2^3} = 1.625$
    - $\frac{17}{4} = 4.25$
    - $\frac{57}{25} = 2.28$
  - If $q \neq 2^m \times 5^n$ → **Non Terminating Repeating** — If Prime Factor of Denominator Contains any Number other than 2 or 5
    - $\frac{29}{6} = 4.8333...$
    - $\frac{1763}{1800} = \frac{1763}{2^3 \times 5^2 \times 3^2} = 0.979444.....$
- **Irrational** — Cannot be Written in p/q Form → **Non Terminating Non Repeating**
  - $12.3450748193...$
  - $\sqrt{2} = 1.414213...$
  - $\pi = 3.1415926...$
  - $\sqrt{\text{NonPerFect Square}}$
  - $\sqrt[3]{\text{NonPerFect Cube}}$

HCF and LCM

1. For any Two Positive Integers a & b
   $HCF(a,b) \times LCM(a,b) = a \times b$

Note : This Formula is Not applicable for More than 2 Numbers

2. $LCM\ of\ Fraction = \frac{LCM\ of\ Numerator}{HCF\ of\ Denominator}$

3. $HCF\ of\ Fraction = \frac{HCF\ of\ Numerator}{LCM\ of\ Denominator}$

4. If One Number is **factor** of other Number, then **Larger Number is LCM** & **Smaller Number is HCF** of two Numbers

Short Tricks Solve Word Problems

HCF → Same Remainder $HCF(N_1 - R, N_2 - R, N_3 - R)$

HCF → Diff Remainder $HCF(N_1 - R_1, N_2 - R_2, N_3 - R_3)$

LCM → Same Remainder $LCM(N_1, N_2, N_3) + R$

LCM → Diff Remainder $LCM(N_1, N_2, N_3) - (N_1 - R_1)$

Condition Applied $(N_1 - R_1) = (N_2 - R_2) = (N_3 - R_3)$

Operation on Rational & Irrational Number

1. $(\text{Rational Number}) \pm \times \div (\text{Rational Number}) = (\text{Rational Number})$
2. $(\text{Rational Number}) \pm \times \div (\text{Irrational Number}) = (\text{Irrational Number})$
3. $(\text{Irrational Number}) \pm \times \div (\text{Irrational Number}) = (\text{Rational or Irrational})$

Ch 7 : Coordinate Geometry

1. Distance Formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

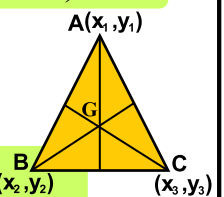
A (x_1, y_1) — B (x_2, y_2)

2. Section Formula $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

A (x_1, y_1) — m — P (x, y) — n — B (x_2, y_2)

3. Mid Point Formula $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

4. Centroid of Triangle $= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$



5. Area of Triangle Formula $= \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$

Collinearity Check

Let there be Three Points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$

(I). Points A,B & C are collinear if **Area of $\triangle ABC = 0$**

(ii). Calculate the Distance between AB, BC & AC. If Sum of Any two distance is equal to 3rd distance, then 3 points are collinear

(iii). Slope Method - Not in Syllabus of CBSE 10th

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_3}{x_2 - x_3}$$ → **Short Trick**

(iv).Simple Observation - Plot the Points on Graph

Short TRICKS

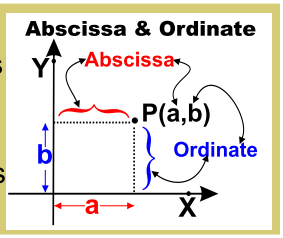
1. **x-axis** Divides **AB** in Ratio= $-y_1 : y_2$
2. **y-axis** Divides **AB** in Ratio= $-x_1 : x_2$
3. Point **P(x,y)** Divides **AB** in Ratio= $-(x - x_1) : (x - x_2)$
4. Point **P(x,y)** Divides **AB** in Ratio= $-(y - y_1) : (y - y_2)$
5. 4th Vertex of ||gram = $(x_1 + x_3 - x_2,\ y_1 + y_3 - y_2)$
6. **Equation of Perpendicular Bisector of Point A & B**

$$(x_1 - x_2)x + (y_1 - y_2)y = \frac{(x_1^2 - x_2^2) + (y_1^2 - y_2^2)}{2}$$

7. **Point on x -axis Equidistant From A & B**
8. **3rd Vertex of Equilateral Triangle**

Absicssa -
Distance of a point from the y-axis (or x-coordinate of a point)

Ordinate -
Distance of a point from the x-axis (or y-coordinate of a point)



Chapter 2 : Polynomial

Polynomial in Simple Terms:

An expression which is composed of **variables, constants and exponents**, that are combined using the mathematical operations such as addition, subtraction, multiplication and division (**Note :** No division by a variable).

Condition for Polynomial :

For the Expression to be called a polynomial ,
i) Power of Variable **must not** be in **Fraction** &
ii) Power of Variable **must not** be **Negative**

Degree : **Highest Power of Variable**

Zeroes : A real number '**k**' is called a zero of the Polynomial **p(x)** , if **p(k) =0**

Geometrical Meaning of Zeroes:

Zeroes of a Polynomial is the **x-coordinates** of Points where Graph intersects the x-axis

Number of Zeroes :

(i) Number of Zeroes of a polynomial is equal to Number of Times the Graph intersects the x-axis
(ii) Number of Zeroes ≤ Degree of Polynomial
(iii) A polynomial of **degree n** has **atmost n zeroes.**

Classification of Polynomials

On The Basis of Degree	On The Basis of Terms
Degree **0** - Constant Degree **1** - Linear Degree **2** - Quadratic Degree **3** - Cubic Degree **4** - Biquadratic (Quartic)	**1 Term** - Monomial $5x^2$, 9x, 4 **2 Unlike Term** – Bionomial $7x+3, 8x^3+x^2$ **3 Unlike Term** - Trinomial $7x + 4 - 5x^2$

Formula of Making Quadratic Polynomial

If α and β are zeroes of Quadratic Polynomial , then
= k[x^2 - (Sum of Zeroes)x + Product of Zeroes]

$= k[x^2 - (\alpha+\beta)x + \alpha\beta]$

$= k(x-\alpha)(x-\beta)$

Formula of Making Cubic Polynomial

= x^3 - (Sum of Zeroes)x^2 + (Sum of Product of Zeroes)x - (Product of Zeroes)

$= x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma$

$= k(x-\alpha)(x-\beta)(x-\gamma)$

Relation b/w Zeroes & Coeff of Quad Poly

$$ax^2+bx+c=0,\ a\neq 0$$

→ **Sum of Zeroes** $(\alpha+\beta) = \dfrac{-\text{Coeff of } x}{\text{Coeff of } x^2} = \dfrac{-b}{a}$

→ **Product of Zeroes** $(\alpha\beta) = \dfrac{\text{Const Term}}{\text{Coeff of } x^2} = \dfrac{c}{a}$

Relation B/w Zeroes & Coeff of Cubic Polyn

$$ax^3+bx^2+cx+d=0,\ a\neq 0$$

→ **Sum of Zeroes** $(\alpha+\beta+\gamma) = \dfrac{-\text{Coeff of } x^2}{\text{Coeff of } x^3} = \dfrac{-b}{a}$

→ **Sum of Prod of Zeroes** $(\alpha\beta+\beta\gamma+\gamma\alpha) = \dfrac{\text{Coeff of } x}{\text{Coeff of } x^3} = \dfrac{c}{a}$

→ **Product of Zeroes** $(\alpha\beta\gamma) = \dfrac{-\text{Const Term}}{\text{Coeff of } x^3} = -\dfrac{d}{a}$

Important Facts

Relation b/w (Sign of Coeff) & (Sign of Roots) of Quadratic Polynomial

	Sign of Roots	a	b	c
Case 1	**Both Roots Positive** $\alpha=+ve,\ \beta=+ve$	+ −	− +	+ −
Case 2	**Both Roots Negative** $\alpha=-ve,\ \beta=-ve$	+ −	+ −	+ −
Case 3	**One Positive, Other Negative** **Mag (+ve Root) > Mag(-ve Root)**	+ −	− +	− +
Case 4	**One Positive, Other Negative** **Mag (-ve Root) > Mag(+ve Root)**	+ −	+ −	− +

Short TRICKS

1. If **a+b+c=0**, Then **x =1,c/a**
2. If **a+c=b**, Then **x=-1, -c/a**
3. Sum of **Reciprocal of Root** = **-b/c**
4. If Roots are Reciprocal of Each Other, Then **a=c**
5. If QP ax^2+bx+c has Roots α & β, Then **QP** Whose Roots are $\frac{1}{\alpha}$ & $\frac{1}{\beta}$ is cx^2+bx+a

Impt NOTE | Multiplying a Polynomial By a Constant, **Does Not affect its Zeroes.**

Common Formulas

1. $(a+b)^3 = a^3+b^3+3ab(a+b)$
2. $(a-b)^3 = a^3-b^3-3ab(a-b)$
3. $(a^3-b^3) = (a-b)(a^2+b^2+ab)$
4. $(a^3+b^3) = (a+b)(a^2+b^2-ab)$
5. $(a+b)^2 = a^2+b^2+2ab$
6. $(a-b)^2 = a^2+b^2-2ab$
7. $a^2-b^2 = (a+b)(a-b)$
8. $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$
9. $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ac)$
10. $a^3+b^3+c^3-3abc = \frac{1}{2}(a+b+c)(a-b)^2(b-c)^2(c-a)^2$
11. $(a+b+c)^3 = a^3+b^3+c^3+3(b+c)(c+a)(a+b)$

Important Formulas

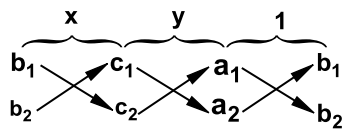
12. $(a-b)^2 = (a+b)^2-4ab$
    $\Rightarrow (a-b) = \sqrt{(a+b)^2-4ab}$
13. $(a+b)^2 = (a-b)^2+4ab$
14. $a^2+b^2 = (a+b)^2-2ab$
15. $a^4+b^4 = [(a+b)^2-2ab]^2-2a^2b^2$

Ch 3 : Linear Eq in 2 Variable

Standard Form $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$

Method of Solving Linear Eq in 2 Variable

1. Graphical Method
2. Algebraic Method
   - (i).Substitution Method
   - (ii).Elimination Method
   - (iii).Cross Multiplication Method



$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Condition for Solvability of Linear Eq

Condition	Number of Solution	Graphical Representation	Graph
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Unique Solution	Intersecting Lines	
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Infinite Many Solution	Coincident Lines	
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	No Solution	Parallel Lines	

Homogeneous System of Equations

$a_1x + b_1y = 0$, $a_2x + b_2y = 0$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Unique Solution (or Zero Solution) (i.e x=0,y=0)	Intersecting Lines	
$\frac{a_1}{a_2} = \frac{b_1}{b_2}$	Infinite Number of Non Zero Solutions	Coincident Lines	

Types of Equation & Their Graph

x=a : Graph is Vertical Line parallel to y-axis
y=b : Graph is Horizontal Line parallel to x-axis
x=y : Graph is Straight Line Passing Through Origin at 45°
x= -y : Graph is Straight Line Passing Through Origin at 135°
ax+by+c=0 : Graph is Straight Line

Short Tricks

1. If **Coefficient of y** & Constant Term is Proportional, Then

$$a_1x + b_1y = c_1$$
$$a_2x + kb_1y = kc_1$$

Then $x = 0$ & $y = \frac{\text{const Term}}{\text{coeff of } y}$

2. If **Coefficient of x** & Constant Term is Proportional, Then

$$a_1x + b_1y = c_1$$
$$ka_1x + b_2y = kc_1$$

Then $y = 0$ & $x = \frac{\text{const Term}}{\text{coeff of } x}$

3. In Equation ax+by+c=0, if **constant term c =0**, then **Line Passes Through Origin**

4. **If x-y = positive Number**
   Then $\frac{1}{x} - \frac{1}{y} =$ **Negative No**

5. **Many Word Problem Short Tricks**

Ch 4 - Quadratic Equation

Defination
An **equation** of the form $ax^2 + bx + c = 0, a \neq 0$ is called a quadratic equation in variable x, where **a, b, c** are constants

Standard Form : $ax^2 + bx + c = 0, a \neq 0$

Method of Solving Quadratic Equation

(i) Factorization Method **(Middle Term Split)**
(ii) Quadratic Formula
(iii) Completing the Square Method **(Removed)**

Quadratic Formula

$$x = \frac{-b \pm \sqrt{D}}{2a}, \text{ where } D = b^2 - 4ac$$

Nature of Roots

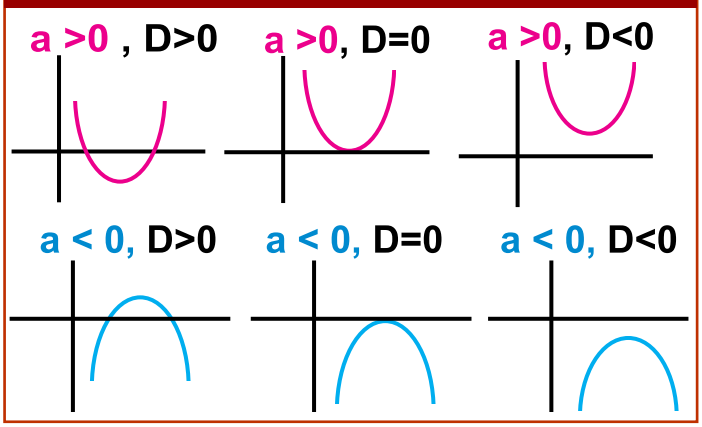
Nature of Root Depends on value of Discriminant(D).

D>0	**Roots are Real and Distinct (Unequal)** If **D** is Perfect Square, The Roots are **Rational** If **D** is not Perfect Square,Then Roots are **Irrational**
D=0	Real & Equal Roots
D<0	No Real Roots (Imaginary Roots)

Note : Condition For Real Root : D≥0

Graph of Quadratic Polynomial (called Parabola)

$$ax^2 + bx + c = 0, \quad a \neq 0$$



Super Important Concept

Quadratic Inequality	If $k^2 < 5^2$	If $k^2 > 5^2$
	Then $-5 < k < 5$	Then $k > 5$ or $k < -5$

Q. Find the value of **k** for which $x^2 + kx + 16 = 0$ has real roots.

This Concept is Not Given in NCERT Book and Questions Based on This Concept had Come in Past in CBSE Exam.

If You Don't Know This Concept, and if Question based on This Come in Exam, you will surely do it wrong

How to Easily Split Middle Term Involving Roots

$$3x^2 - 4\sqrt{3}x + 4 = 0$$

Amazing Short Trick

Ch 5 - Arithmetic Progression

Arithmetic Progression (AP)

It is a **sequence** of numbers,in which the **difference** between any two consecutive numbers is **constant.**

Difference b/w Progression, Sequence, Series

Both **Progressions & Sequence** are sets of numbers which are arranged in a definite order according to some definite rule.

The **difference between a progression and a sequence** is that a **Progression** has a Specific Formula to calculate its nth term, whereas a **Sequence** can be based on a logical rule like 'a group of prime numbers'

Thus, every progression is a sequence but every sequence is not a Progression.

Series

A series is a sequence of numbers that is added by + signs. The word 'series' is said to represent the sum of the numbers, and not the sum itself

Important Formulas

1. $T_n = a + (n-1)d$
2. $T_n = S_n - S_{n-1}$
3. nth Term From End $= l - (n-1)d$
4. $S_n = \frac{n}{2}(2a + (n-1)d)$
5. $S_n = \frac{n}{2}(a + l)$

Where
a= First Term,
d= Common Difference,
n=Number of Terms
Tn= value of nth Term,
Sn= Sum of n Terms
l=Last Term

Selection of Terms in AP

3 Terms in AP : (a-d),a,(a+d)
4 Terms in AP : (a-3d),(a-d),(a+d),(a+3d)
5 Terms in AP : (a-2d),(a-d),a,(a+d),(a+2d)

Condition for 3 Terms to be in AP

Three quantities **a, b, c** will be in **AP**, if the **difference b/w two successive terms** are **equal**, i.e., **b – a = c – b or 2b = a + c,**

Some Standard Results

1. If $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, Then $T_{mn} = 1$
2. If $mT_m = nT_n$, Then $T_{(m+n)} = 0$
3. If $T_m = n$ & $T_n = m$, Then $T_{(m+n)} = 0$
4. If $T_p = q$ and $T_q = p$, Then $T_n = (p + q - n)$
5. If $T_p = a$, $T_q = b$, & $T_r = c$, Then $a(q-r) + b(r-p) + c(p-q) = 0$
6. If $S_m = S_n$, Then $S_{(m+n)} = 0$
7. If $S_m = n$ & $S_n = m$, Then $S_{(m+n)} = -(m+n)$
8. If $T_m = \frac{1}{n}$ & $T_n = \frac{1}{m}$, Then $S_{mn} = \frac{(1+mn)}{2}$
9. If $\frac{S_m}{S_n} = \frac{m^2}{n^2}$, Then $\frac{T_m}{T_n} = \frac{2m-1}{2n-1}$

Middle Term in AP

If Number of Terms is **Odd**, Then There is One Middle Term
Middle Term $= \frac{n+1}{2}$

If Number of Terms is **Even**, Then There are Two Middle Terms
Middle Term $= \left(\frac{n}{2}, \frac{n}{2}+1\right)$

Arithmetic Mean :

If **3** numbers **a,b,c** are in AP, Then **Middle Number 'b'** is called **Arithmetic Mean** of a & c

Important Facts / Short Tricks

1 **nth term of AP is a Linear Expression in n.**
nth term (Tn) = an+b,
Where "**coefficient of n**" is **common difference** & (a+b) is **First Term** of the AP
For Example : nth term =7n+3
Then, **First Term** = (7+3)=10 & **c.d**=7

2 **Sum of 'n' Term of AP is Quadratic Expression in n**
$S_n = An^2 + Bn$, **Where**
Common Difference = 2A & First Term = (A+B)

3 Sum of Terms **Equidistant from Beginning & End** of an AP is always constant &
is equal to the sum of First Term & Last Term (a+*l*)
or in other words
Sum of **nth term from beginning** & **nth term from end** is always constant and is equal to (a+*l*)

4 If the **ratio of Sum of First n Term of two AP's** are given and we have to find the **Ratio of mth Term**, then Replace n by **(2m-1)** in the original given ratio.
For Example
$\frac{S_n}{s_n} = \frac{7n+1}{4n+27}$, then $\frac{T_m}{t_m} = \frac{7(2m-1)+1}{4(2m-1)+27}$

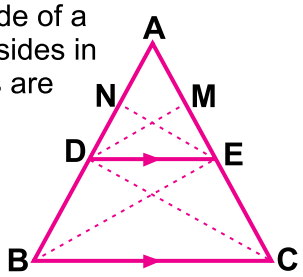
Useful Shortcut (Memorize it)

1. **Sum of First n Natural Number :**
$1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$
2. **Sum of First n Odd Natural Numbers :**
$1 + 3 + 5 + ... + (2n-1) = n^2$
3. **Sum of First n Even Natural Numbers :**
$2 + 4 + 6 + ... + 2n = n(n+1)$

Chapter 6 - Triangles

1. Thales Theorem
(or BPT- Basic Proportionality Theorem)

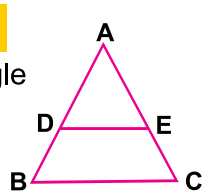
If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.



In Short,
If $DE \parallel BC$, Then $\frac{AD}{BD} = \frac{AE}{CE}$

2. Converse of BPT Theorem

If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

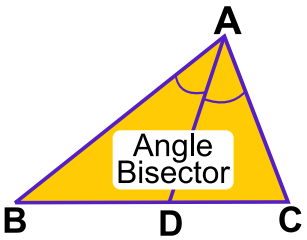


In Short, If $\frac{AD}{BD} = \frac{AE}{CE}$, Then $DE \parallel BC$

3. Angle Bisector Theorem

Internal Bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

In Short,
If AD is bisector of $\angle A$,
Then $\frac{AB}{AC} = \frac{BD}{DC}$

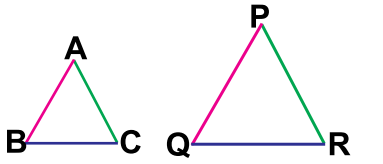


4. Ratio of Areas of Similar Triangles

Theorem 1. The Ratio of areas of two similar triangles is equal to the ratio of the squares of thier **corresponding sides.**

If $\Delta ABC \sim \Delta PQR$, Then

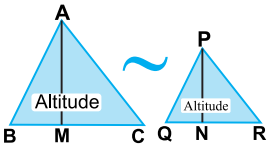
$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$$



Theorem 2. The ratio of the areas of two similar triangles is equal to the ratio of the squares of their **corresponding Altitudes.**

If $\Delta ABC \sim \Delta PQR$, Then

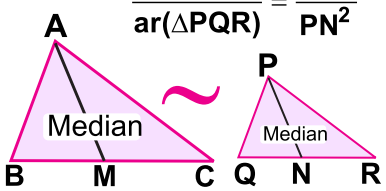
$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AM^2}{PN^2}$$



Theorem 3. The ratio of areas of two similar triangles is equal to the ratio of the squares of their **corresponding Medians**

If $\Delta ABC \sim \Delta PQR$, Then

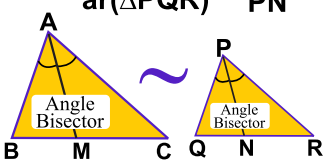
$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AM^2}{PN^2}$$



Theorem 4. The ratio of areas of two similar triangles is equal to the ratio of the squares of their **corresponding Angle Bisector**

If $\Delta ABC \sim \Delta PQR$, Then

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AM^2}{PN^2}$$

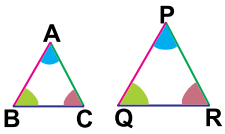


Similar Triangles

If two triangles are similar then their corresponding angles are equal & corresponding sides are proportional

i.e If $\Delta ABC \sim \Delta PQR$, Then

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \quad \& \quad \angle A = \angle P,\ \angle B = \angle Q,\ \angle C = \angle R$$

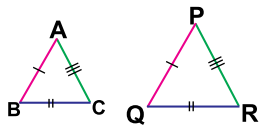


Note : Simiar Triangles have Same Shape, But Not Necessarily the Same Size

Criteria for Similarity of Two Triangles

SSS Similarity

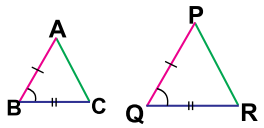
If the three sides of one triangle are respectively proportional to the corresponding three sides of another triangle, then they are similar.



$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

SAS Similarity

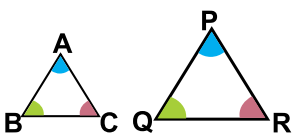
If two sides of a triangle are respectively proportional to the corresponding sides of another triangle and included angle are equal then the triangles are similar.



$$\frac{AB}{PQ} = \frac{BC}{QR} \ \& \ \angle B = \angle Q$$

AAA Similarity or (AA)

If the three angles of one triangle are respectively equal to the corresponding angles of another triangle, then the two triangles are similar.

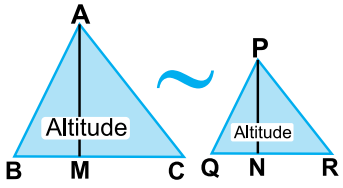


$$\angle A = \angle P, \quad \angle B = \angle Q, \quad \angle C = \angle R$$

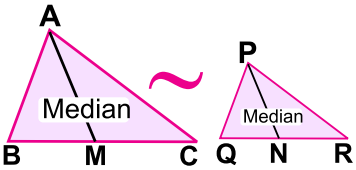
Properties of Similar Triangles

If two triangles are similar, then ratio of **corresponding sides** is equal to the

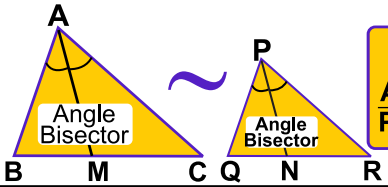
- ratio of corresponding **altitudes** or
- ratio of corresponding **medians** or
- ratio of corresponding **angle Bisector.**



If $\Delta ABC \sim \Delta PQR$, Then $\frac{AB}{PQ} = \frac{\text{Altitude AM}}{\text{Altitude PN}}$



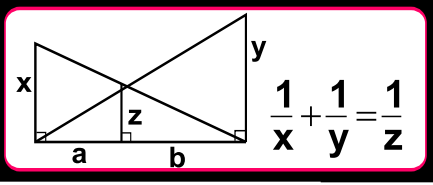
If $\Delta ABC \sim \Delta PQR$, Then $\frac{AB}{PQ} = \frac{\text{Median AM}}{\text{Median PN}}$



If $\Delta ABC \sim \Delta PQR$, Then $\frac{AB}{PQ} = \frac{\text{Angle Bisector AM}}{\text{Angle Bisector PN}}$

If $\Delta ABC \sim \Delta DEF$, Then

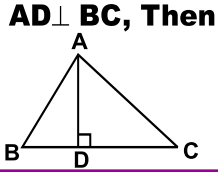
$$\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$



$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

In ΔABC, if $\angle A = 90$ & $AD \perp BC$, Then

(i). $AD^2 = BD.CD$
(ii). $AB^2 = BD.BC$
(iii). $AC^2 = CD.CB$



Ch 8 : Trigonometry

Trigonometry is the Branch of Mathematics in which we study the relationship between the sides & angles of a triangle.

One Trigonometric Ratio in terms of Other T-Ratio

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$
$\sin\theta$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\operatorname{cosec}\theta}$
$\cos\theta$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$
$\tan\theta$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\cot\theta$	$\frac{\sqrt{(1-\sin^2\theta)}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{(1-\cos^2\theta)}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sqrt{(\sec^2\theta-1)}}$	$\sqrt{(\operatorname{cosec}^2\theta-1)}$
$\sec\theta$	$\frac{1}{\sqrt{(1-\sin^2\theta)}}$	$\frac{1}{\cos\theta}$	$\sqrt{(1+\tan^2\theta)}$	$\frac{\sqrt{(1+\cot^2\theta)}}{\cot\theta}$	$\sec\theta$	$\frac{\operatorname{cosec}\theta}{\sqrt{(\operatorname{cosec}^2\theta-1)}}$
$\operatorname{cosec}\theta$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{(1-\cos^2\theta)}}$	$\frac{\sqrt{(1+\tan^2\theta)}}{\tan\theta}$	$\sqrt{(1+\cot^2\theta)}$	$\frac{\sec\theta}{\sqrt{(\sec^2\theta-1)}}$	$\operatorname{cosec}\theta$

Values of T Ratios

	0°	30°	45°	60°	90°
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ & $\cot\theta = \frac{\cos\theta}{\sin\theta}$

NOTE : $\sin^2\theta \neq \sin\theta^2$

But $\sin^2\theta$ is same as $(\sin\theta)^2$

$\tan1^\circ \tan2^\circ \tan3^\circ \ldots \tan89^\circ = 1$

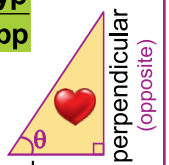
$\cos1^\circ \cos2^\circ \cos3^\circ \ldots \cos90^\circ = 0$

Trigonometry Ratios

(i) $\sin\theta = \frac{p}{h} = \frac{opp}{hyp}$, (iv) $\operatorname{cosec}\theta = \frac{h}{p} = \frac{hyp}{opp}$

(ii) $\cos\theta = \frac{b}{h} = \frac{adj}{hyp}$, (v) $\sec\theta = \frac{h}{b} = \frac{hyp}{adj}$

(iii) $\tan\theta = \frac{p}{b} = \frac{opp}{adj}$, (vi) $\cot\theta = \frac{b}{p} = \frac{adj}{opp}$



Reciprocal Relation

(i) $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$, $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

(ii) $\cos\theta = \frac{1}{\sec\theta}$, $\sec\theta = \frac{1}{\cos\theta}$

(iii) $\tan\theta = \frac{1}{\cot\theta}$, $\cot\theta = \frac{1}{\tan\theta}$

Complementary Angle

$\sin(90-\theta) = \cos\theta$

$\cos(90-\theta) = \sin\theta$

$\tan(90-\theta) = \cot\theta$

$\cot(90-\theta) = \tan\theta$

$\operatorname{cosec}(90-\theta) = \sec\theta$

$\sec(90-\theta) = \operatorname{cosec}\theta$



$-1 \le \sin\theta \le 1$

$-1 \le \cos\theta \le 1$

Trigonometry Identities

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$\operatorname{cosec}^2\theta - 1 = \cot^2\theta$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\operatorname{cosec}\theta - \cot\theta = \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

$\operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}$

$\sec^2\theta - 1 = \tan^2\theta$

$\sec^2\theta = 1 + \tan^2\theta$

$\sec^2\theta - \tan^2\theta = 1$

$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$

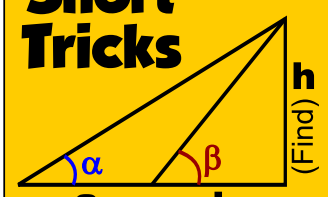
$\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$

Useful Shortcuts

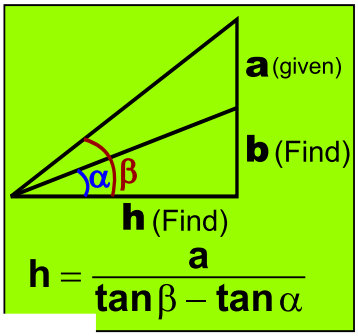
1. $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$
2. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$
3. $\sin^8\theta + \cos^8\theta = 1 + 2\sin^4\theta\cos^4\theta - 4\sin^2\theta\cos^2\theta$
4. If $\sin\theta + \operatorname{cosec}\theta = 2$, then $\sin^n\theta + \operatorname{cosec}^n\theta = 2$
5. If $\cos\theta + \sec\theta = 2$, then $\cos^n\theta + \sec^n\theta = 2$
6. If $\tan\theta + \cot\theta = 2$, then $\tan^n\theta + \cot^n\theta = 2$
7. If $a\sin\theta + b\cos\theta = c$, Then $a\cos\theta - b\sin\theta = \pm\sqrt{a^2+b^2-c^2}$
8. If $a\sin\theta - b\cos\theta = c$, Then $a\cos\theta + b\sin\theta = \pm\sqrt{a^2+b^2-c^2}$
9. If $a\cos\theta + b\sin\theta = c$, Then $a\sin\theta - b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$
10. If $a\cos\theta - b\sin\theta = c$, Then $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$
11. If $\sin\theta + \cos\theta = c$, Then $\sin\theta - \cos\theta = \pm\sqrt{2-c^2}$
12. If $\sec\theta + \tan\theta = p$, Then $\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$
13. If $\sec\theta - \tan\theta = p$, Then also $\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$
14. If $\operatorname{cosec}\theta + \cot\theta = p$, Then $\operatorname{cosec}\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$
15. If $\operatorname{cosec}\theta - \cot\theta = p$, Then also $\operatorname{cosec}\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

Ch 9 : Height & Distance

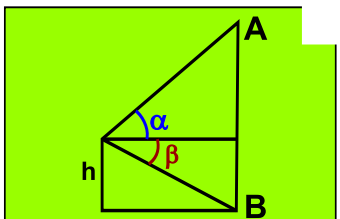
Short Tricks



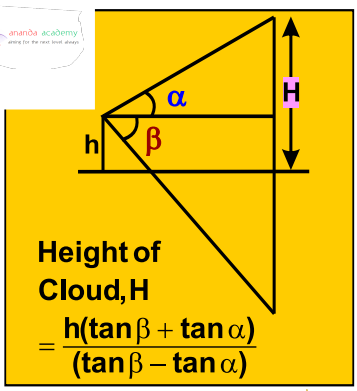
$h = \frac{a\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$



$h = \frac{a}{\tan\beta - \tan\alpha}$

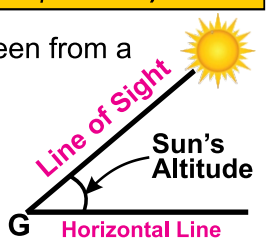


Height of Houe, AB $= h(1 + \tan\alpha\cot\beta)$



Height of Cloud, H $= \frac{h(\tan\beta + \tan\alpha)}{(\tan\beta - \tan\alpha)}$

Sun's Altitude : If the sun is seen from a point G then the angle between **Horizontal Line** & **Line of Sight** is called Sun's Altitude. Thus Angle of Elevation of the Sun from a point P is called Sun's Altitude.



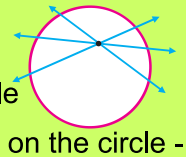
Ch 10 : Circles

Circle is collection of all the points which are equidistant from a fixed Point called Centre.

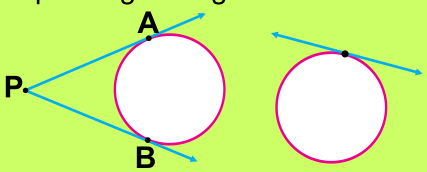
Tangent : It is a line which intersect the circle at only one point. The Point at which the tangent line intersects the circle is called the **point of contact.**

Number of tangents to a Circle :

(i) When the point lies inside the circle
- No Tangent Possible



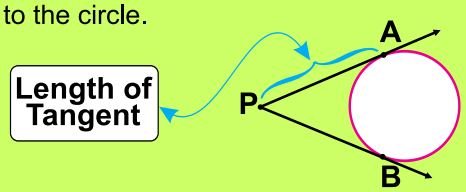
(ii) When the point lies on the circle - There is one & only one tangent passing through that Point



(iii) When point lies outside the circle - There are exactly two tangents Possible.

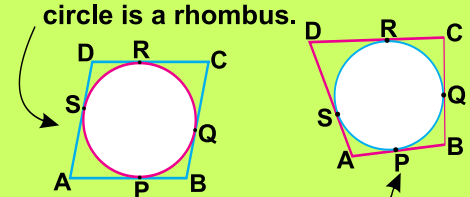
Length of a Tangent :

the length of the segment of the tangent from the external point P to the point of contact with the circle is called the length of the tangent from the point P to the circle.



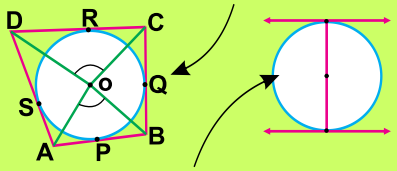
Some Standard Results of Circle

(I) Parallelogram circumscribing a circle is a rhombus.



(ii) If a quadrilateral ABCD is drawn to circumscribe a circle.
Then **AB + CD = AD + BC**

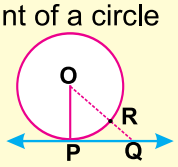
(iii) Opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at centre of circle. i.e **∠AOB + ∠COD=180°**



(iv) Tangents drawn at the ends of a diameter of a circle are parallel.

Theorem 1.

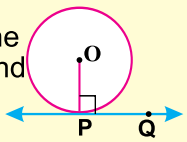
The tangent at any point of a circle is perpendicular to the radius through the point of contact



i.e If PQ is Tangent & OP is Radius, then ∠OPQ=90°

Theorem 2.

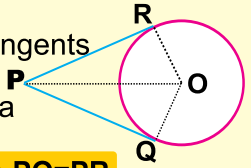
A line drawn through the end point of a radius and perpendicular to it, is a tangent to the circle



i.e If OP is Radius & ∠OPQ=90° then PQ is Tangent

Theorem 3.

The lengths of tangents drawn from an external point to a circle are equal.

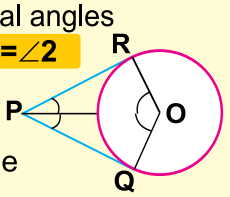


i.e PQ=PR

Theorem 4.

If two tangents are drawn to a circle from an external point then
(I) they subtend equal angles at the centre. **i.e ∠1=∠2**

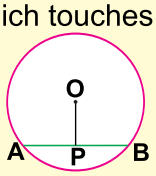
(ii) they are equally inclined to the line segments joining the centre to that point.



i.e ∠3=∠4

Theorem 5.

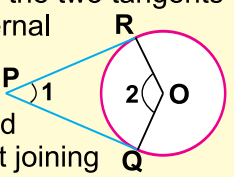
In two concentric circles,the chord of the larger circle,which touches the smaller circle, is bisected at the point of contact.



i.e AP=PB

Theorem 6.

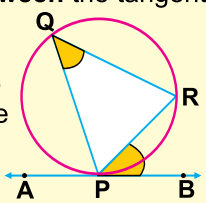
The angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.



i.e ∠1+∠2=180°

Alternate Segment Theorem

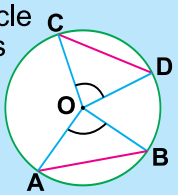
The **angle formed between** the tangent and the chord through the point of contact of the tangent **is equal to** the angle formed by the chord in the alternate segment".



The alternate segment theorem is also known as the **tangent-chord theorem.**

Class 9 Theorems
useful in class 10

1. Equal chords of a circle subtend equal angles at the centre.

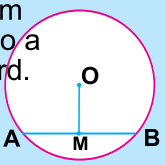


i.e if AB=CD, Then ∠AOB =∠COD

2. If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

i.e if ∠AOB =∠COD, Then AB = CD

3. The perpendicular from the centre of a circle to a chord bisects the chord.

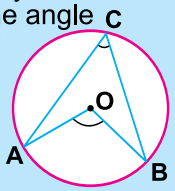


i.e if OM ⊥ AB, Then, AM = BM

4. The line joining the centre of a circle & mid point of chord is perpendicular to the chord.

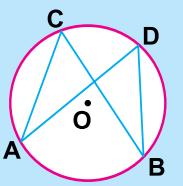
if AM=BM, Then OM ⊥ AB

5. The angle subtended by an arc at the centre is double the angle subtended by it at any point on remaining part of the circle.



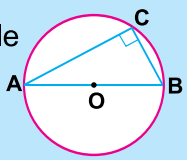
∠AOB =2∠COD

6. Angles in the same segment of a circle are equal.



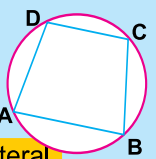
i.e ∠ACB =∠ADB

7. Angle in a semicircle is a right angle.



i.e ∠ACB = 90°

8. The sum of either pair of opposite angles of a cyclic quadrilateral is 180°

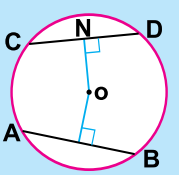


If ABCD is cylic Quadrilateral
Then, ∠A+∠B = 180°

9. If the sum of a pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.

If, ∠A+∠B = 180°
Then ABCD is cylic Quadrilateral

10. Equal chords of a circle are equidistant from the centre.

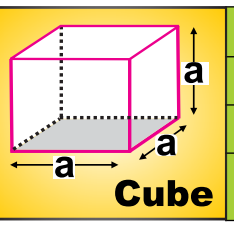


i.e If AB=CD, Then OM=ON

11. Chords equidistant from the centre of a circle are equal in length.

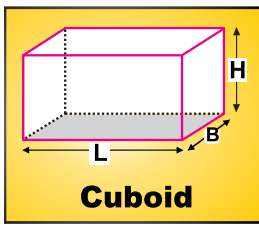
i.e If OM=ON, Then AB=CD

Ch 12 – Surface Area & Volume



Cube

LSA	$4(\text{Side})^2$
TSA	$6(\text{Side})^2$
Volume	$(\text{Side})^3$
Diagonal of Cube	$\text{side}\sqrt{3}$



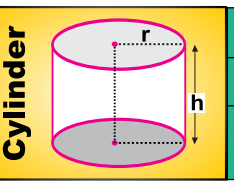
Cuboid

LSA	$2(L+B)\times H$
TSA	$2(LB+BH+LH)$
Volume	$L\times B\times H$
Diagonal of Cuboid	$\sqrt{L^2+B^2+H^2}$

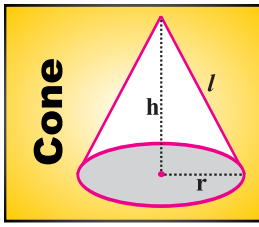
kilo
hecto
deca
meter
deci
centi
mili

Common Pythagorean Triplets

3,4,5
5,12,13
8,15,17
9,40,41
20,21,29



CSA	$2\pi rh$
TSA	$2\pi rh+2\pi r^2$
Volume	πr^2h

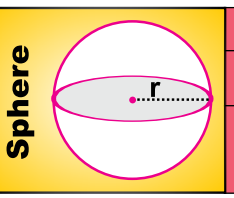


CSA	πrl
TSA	$\pi rl+\pi r^2$
Volume	$\frac{1}{3}\pi r^2h$
Slant Height	$l=\sqrt{r^2+h^2}$

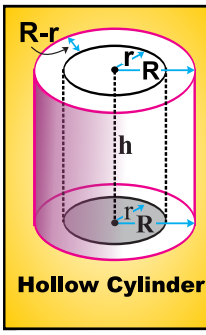
Volume Conversion

$1m^3$ = 1000 Litre
$1cm^3$ = 1/1000 Litre
$1000cm^3$ = 1 Litre
1 Hectare = $10000\ m^2$

Note :
Litre is not Equal to kg
Litre is Unit of Volume & Kg is Unit of Mass



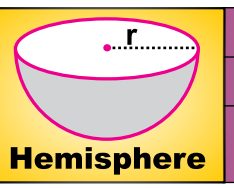
CSA	.....
TSA	$4\pi r^2$
Volume	$\frac{4}{3}\pi r^3$



Hollow Cylinder

1. Outer CSA $= 2\pi Rh$
2. Inner CSA $= 2\pi rh$
3. Area of 2 Ring $= 2\pi\,(R^2-r^2)$
4. Total Surface Area (TSA) $= 2\pi rh+2\pi Rh+2\pi(R^2-r^2)$ a
5. Volume of Solid Portion $=\pi(R^2-r^2)\times h$
6. Thickness $=(R-r)$

To Convert
m/sec into km/hr
Multiply By **18/5**
For Example: 10m/sec
$=10\times\frac{18}{5}=36\text{km/hr}$



CSA	$2\pi r^2$
TSA	$3\pi r^2$
Volume	$\frac{2}{3}\pi r^3$

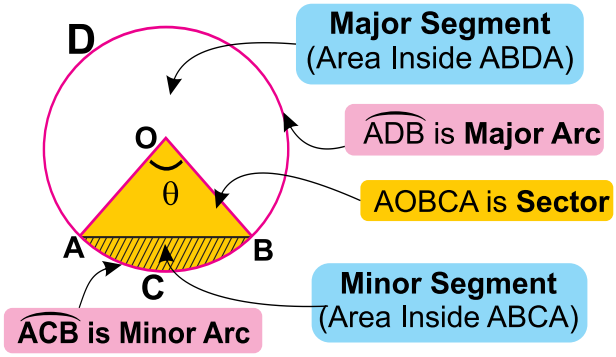
STOP

Mass or Weight = Volume x Density
Volume of Prism = Area of Base x Height

Ch 11 – Area Related to Circle

Square Root Short Trick

$$\sqrt{k}=\sqrt{x\pm y}=\sqrt{x}\pm\frac{y}{2\sqrt{x}}$$

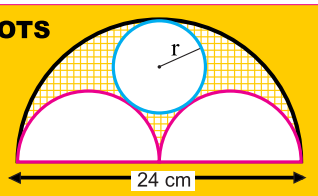


1. Arc Length $=\frac{\theta}{360}\times 2\pi r$
2. Area of Sector of Circle $=\frac{\theta}{360}\times\pi r^2$
   $=\frac{1}{2}\times$ **Radius** $\times$ **Arc Length (Impt)**
3. Area of Minor Segment
   $=$ Area of Sector $-$ Area of Triangle
   $=\frac{\theta}{360}\pi r^2-\frac{1}{2}r^2\sin\theta$
4. Area of Major Segment
   $=$ Area of Circle $-$ Area of Minor Segment
5. Area of Triangle $=\frac{1}{2}r^2\sin\theta$
6. **Number of Rotation Per Minute**
   $=\frac{\text{Distance Covered by Wheel in 1 Minute}}{\text{Circumference of Wheel}}$

Common Formulas

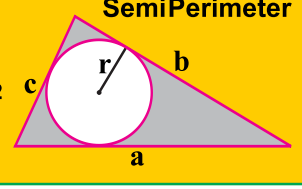
7. Area of Circle $=\pi r^2$
8. Circumference of Circle $=2\pi r$
9. Area of Semi Circle $=\frac{\pi r^2}{2}$
10. Perimeter of Semicircle $=\pi r+2r$
11. Area of Quadrant $=\frac{\pi r^2}{4}$
12. Perimeter of Quadrant $=\frac{2\pi r}{4}+2r$
13. Area of Square $=(\text{Side})^2=\frac{1}{2}(\text{Diagonal})^2$
14. Area of Rectangle $=L\times B$
15. Diagonal of Square $=\text{Side}\sqrt{2}$
16. Diagonal of Rectangle $=\sqrt{L^2+B^2}$
17. Area of Rhombus $=\frac{1}{2}\times d_1\times d_2$ (or) Base $\times$ Height
18. Area of Parallelogram $=$ Base $\times$ Height
19. Area of Equilateral Triangle $=\frac{\sqrt{3}}{4}(\text{side})^2$
20. Height of Equilateral Triangle $=\frac{\sqrt{3}}{2}\times\text{Side}$
21. Area of Triangle $=\frac{1}{2}\times\text{Base}\times\text{Height}=\sqrt{s(s-a)(s-b)(s-c)}$
22. Area of Trapezium $=\frac{1}{2}(\text{Sum of Parallel Sides})\times\text{Height}$

HOTS



Radius (r)
=1/6 Diameter of Outer Semicircle

Inradius $=\frac{\text{Area of Triangle}}{\text{Semi Perimeter}}$



23. Area of 4 Wall $=2(L+B)\times H$

$\sqrt{2}=1.414$
$\sqrt{3}=1.732$
$\sqrt{5}=2.236$

Ch 13 – Statistics

Mean : The Average of All Numbers

Median : is the middle most value in a given data set after it is arranged in ascending order.

Mode : The Number that appears the Most in Given Data

Mean

M-1 Direct Method, $\bar{X} = \frac{\sum fixi}{\sum fi}$

M-2 Assumed Mean Method, $\bar{X} = A + \frac{\sum fidi}{\sum fi}$

M-3 Step Deviation Method, $\bar{X} = A + \left(\frac{\sum fiui}{\sum fi}\right) \times h$

where,

A = Assumed Mean, x_i = Class Mark

$d_i = x_i - A$, $u_i = \left(\frac{x_i - A}{h}\right)$

h = Class Size = Upper Limit – Lower Limit

$\text{Class Mark} = \left(\frac{\text{Lower Limit} + \text{Upper Limit}}{2}\right)$

Median

$$= L + \left(\frac{\frac{n}{2} - pcf}{f}\right) \times h$$

Mode

$$= L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

where,

L = lower limit of **Modal/Median** Class
n = Sum of Frequencies
h = **width** of class interval (or Class Size)
f = **Frequency** of Median Class
pcf = **Cumulative Frequency** of Class Preceding Median Class
f1 = Frequency of **Modal Class**
f0 = Frequency of class **Preceding to Modal Class**
f2 = Frequency of class **suceeding to Modal Class**

Median of Ungrouped Data

If n is Odd → $\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ term}$

If n is Even → $\text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}}{2}$

Steps to Calculate Median of Grouped Data

Step 1 : Calculate the **cumulative frequency** of each class.
Step 2 : Calculate **n/2** (n is sum of all Frequencies)
Step 3 : Then find the class in which the value of cf is **just greater than n/2**. This is the Median Class.
Step 4 : After Identifying Median Class, Obtain Values of **L, h, f, pcf**
Step 5 : Put All Values in Formula & Calculate

Emperical Relation bw Mean Median & Mode
Mode = 3Median - 2Mean

Ch 14 – Probability

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total Number of Possible Outcomes}} = \frac{n(E)}{n(S)}$$

- **Probability :** how likely something is to happen. Probability is a concept which numerically measures the degree of certainity of the occurrence of Events
- **Experiment :** An operation which can produce some well defined outcomes is called an experiment.
- **Random Experiment :** An experiment in which all possible outcomes are known, and the exact outcome cannot be predicted in advance, is called a random experiment.
- **Deterministic Experiment :** An experiment whose outcome may be predicted with certainty beforehand
- **Sample Space :** Set of all possible outcomes of a random experiment.
- **Event :** The collection of all or some of the possible outcomes is called an event
- **Elementary Event :** If a random experiment is performed, then each of its outcomes is known as an elementary event. **The Sum of Probabilities of all the elementary events of an experiment is 1**
- **Equally Likely Events :** Those events which have an equal probability of occuring. For example: Each numeral on a die is equally likely to occur when the die is tossed.
- **Complementary Event :** **Two events are said to be complementary when one event will occur if and only if the other does not. The event (not E) is called the complementary event of E. We Denote (not E) by E′ or $\bar{E}$**
The Sum of probabilities of two complimentary events is 1.

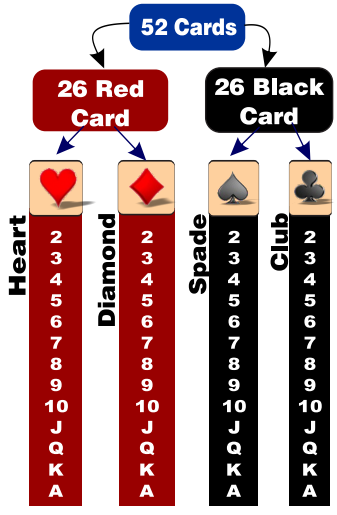
Sample Space when Two Dice are Thrown

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

2 Coins
HH, HT
TH, TT

3 Coins
H H H
H H T
H T H
H T T
T H H
T H T
T T H
T T T

P(Sure Event) =1
P(Impossible Event) =0
$0 \le P(E) \le 1$
$P(\bar{E}) = 1 - P(E)$
$P(E) + P(\bar{E}) = 1$
$P\left(\begin{matrix}53 \text{ Sunday} \\ \text{in Normal Year}\end{matrix}\right) = \frac{1}{7}$
$P\left(\begin{matrix}53 \text{ Sunday} \\ \text{in Leap Year}\end{matrix}\right) = \frac{2}{7}$



No. of Face Card = 12
(4 Jack, 4 Queen, 4 King)



When 3 Coins are Tossed

P (One Head)	=3/8
P (Two Head)	=3/8
P (Three Head)	=1/8
P (at Least One Head)	=7/8
P (at Least Two Head)	=4/8
P (at Least Three Head)	=1/8
P (at most One Head)	=4/8
P (at most Two Head)	=7/8
P (at most Three Head)	=8/8